Critical issues in mineral resources appraisal
矿产资源评价中的核心问题

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Abstract: Mineral resources appraisal has been an important research subject for geoscientists around the world in the past three decades. Significant progress has been made in development of statistical techniques and estimation methodologies for mineral resources. Integration of multiple data sets, either by experts or statistical methods, has become a common practice in estimation of mineral potentials. However, real effect of these methodologies is at best very limited in term of uses for government macro policy making, resource management, and mineral exploration in commercial sectors. Several major problems in data integration remain to be solved in order to achieve significant improvement in the effect of resource estimation. Some critical issues, considered being critical and fundamental in mineral resources appraisal, are addressed in this paper. Issues related to randomness of mineral endowment, intrinsic statistical relations, exceptionalness of ore, intrinsic geological units, and economic translation and truncation, are discussed in this paper. Moreover, a number of specific important technical issues in information synthesis are also identified as well, including information enhancement, spatial continuity, data integration and target delineation. Finally, a new concept of dynamic control areas is proposed for future development of quantification of mineral resources.

Key words: intrinsic geological unit; geological processes; mineral resource appraisal; information synthesis; mineral target; dynamic control areas

摘要：矿产资源评价是过去几十年里全球地质学家关注的重要领域。在过去的研究中，矿产资源评价的方法学及其相应的统计学技术在理论上取得了明显的进展，统计学技术或主观评估已经成为多元数据综合和资源潜力预测的通用方法。然而，这些方法所得出的估计结果或结论对于国家宏观政策制定、矿产资源管理、商业矿产勘探的实际效果和指导意义非常有限。影响矿产资源评价效果的很多问题还有待于进一步研究，新的基础理论问题需得到进一步解决。为此，就其中几个有关矿产资源评价的最为关键的问题进行了深入的探讨，包括矿产资源的随机性、内在的统计关系、矿产的稀有性、本征地质单元、资源经济转化等。就几个信息综合中的技术问题也做了较为系统的讨论，包括信息增强、空间连续性、数据综合及靶区圈定。最后提出了"动态模型区"的概念，试图为矿产资源评价指出一个新的思考方向。

关键词：本征地质单元；地质过程；矿产资源评价；信息综合；矿产靶区；动态模型区
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Some fundamental issues in mineral resources assessment were systematically addressed[1]. Some of the critical issues are reconsidered here with respect to new understanding of basic geo−relations between resource descriptors and geological processes. Various multivariate models and techniques have been used

over the past two decades to relate geological variables to some aspects of mineral occurrence or deposits. Conventional objective methods for mineral resource assessment have estimated either mineral endowment or discoverable mineral resources of a particular type of deposit in a region. The mineral endowment of a region usually refers to that quantity of mineral in accumulations meeting specified physical characteristics, such as grade, size, and depth. A multivariate endowment model is essentially characterized by a particular information extraction strategy for the so-called optimum combination of those geological features most related to spatial variations of endowment[2]. Most of these models estimate mineral resources based upon the principle of analogy, i.e., the resources in a study region are estimated by a model that is established on a control area by assuming different regions with similar geological environments have similar endowment[2-6].

Most of these models have employed as information reference a grid of regularly spaced cells(inter-grid areas) and have dealt in one way or another with either mineral favorability, probability, mineral wealth or density of mineral occurrence (deposit). Of special interests have been those models that describe uncertainty about these estimates, such as the probability for occurrence of mineral deposits within a cell. These studies seem to have been a necessary step in the evolution of the science of mineral resources prediction, because geologists in general have been slow to adapt quantitative methods, and even more slow to substitute objective and quantitative analysis for all or part of subjective analysis. Thus, there was a need to demonstrate quantitative methods that could be used to estimate undiscovered mineral resources. However, to some extent, this reluctance represented the dissatisfaction by geologists for the at-best low, and sometimes trivial, level of geoscience information captured by the quantitative variables and related to mineral occurrence by the multivariate models. Simply stated, mineral resource estimates by quantitative and objective methods will not improve significantly until more geoscience information is related in more appropriate ways to the various descriptors of mineral resources.

The concept of mineral resource is many faceted, including physical and chemical properties of mineral deposits, as they occur naturally in the earth´s crust and economic properties created by man´s socio—technical production system and the demands for mineral materials derived there from. The discussion presented here focuses upon several aspects of mineral resources that are fundamental considerations in the effective information synthesis for mineral resource estimation: randomness of mineral endowment, basic statistical relations, scarceness, geological foundations, economic truncation and translation, and spatial continuity. Some major issues in quantitative mineral resource estimation are addressed, including information enhancement, information synthesis, as well as target identification. Information synthesis is a central task in both mineral exploration and resource estimation.

1   Randomness of mineral endowment

Most of the past and current studies on mineral resource estimation have been constructed and applied on the basis of a common assumption that mineral endowment descriptors and at least some of the related geologic processes behave more or less according to certain stochastic rules. The assumption is seldom challenged, although controversies have continued over four decades, for example, the types of the stochastic laws that govern the true distributions of geochemical element concentrations[3,7-8]. This seems to indicate that the assumption that some geological processes are to some extent stochastic and follow certain stochastic laws have been widely accepted, although it is premature to assert that all of the geoscience features are stochastic. It is useful to examine this notion before investigating specific stochastic laws for particular geologic events, the use of statistical models to estimate mineral resources, and probabilistic descriptions of resource descriptors.

In his famous "Ideal Granite Model", Vistelius[9] showed that the crystallization of minerals, such as potassium feldspar, quartz, as well as plagioclase con-

tained in the "ideal granite" can be modeled by some stochastic functions that vary in space and time. It has been proved mathematically that there is a three-dimensional "packing of particles" such that the three mutually perpendicular directions can be described according to the Markov property in each direction with identical transition probability matrices in the three directions[10]. Another example due to Vistelius is his gravitational stratification package model[11]. In the study of red beds of the Cheleken Peninsula, under certain assumptions, Vistelius showed that the sequence of red beds with two distinct states, S (arenaceous beds) and A (argillaceous beds), which can be treated as a homogenous reversible Markov chain of second order, with the partial transition through A being first order Markov and the partial transitions through S being second-order Markov.

Sedimentary sequences have been regarded generally as some types of cyclic processes which are associated with certain Markov properties[12–15]. Pan[14] demonstrated that many sedimentary sections can be treated as homogeneous stochastic processes if no significant depositional discontinuities or structural unconformities occur in the sequences and that homogeneous sedimentary processes can be decomposed uniquely into the sum of independent reversible and unidirectional stochastic flows.

The process of ore deposition was closely examined by Kantsel[16] based upon the function of metal distribution in ores. The process of hydrothermal mineralization during a single stage can be treated as a continuous stationary process of the Markov type. The resulting concentration of metal can be represented by a distribution function, a most important characteristic reflecting the speed of the mineralization process.

These contributions are informative about some fundamental and crucial controversial issues regarding the application of stochastic models to mineral exploration, although some concerns cannot be satisfactorily resolved without more research. A partial conclusion drawn from these preliminary works should be that at least under certain conditions some of the geologic or earth processes can be modeled by stochastic laws. However, it would be incorrect to associate the earth processes with the stochastic laws through one to one relations, since the random properties of geologic events generally are space and time dependent.

2　Fundamental geo-process relations

Observations on geologic features in certain spatial and temporal settings are the outcomes of a sequence of geologic processes superimposed during crustal evolution and initiated by inner energies of the earth, biosphere, hydrosphere, atmosphere, as well as other universal forces. Conceptually, there should be two levels of cause-effect relations among the geologic events, crustal evolution and initial forces, that created the earth. The earth commonly represents the entity of earth processes, e.g., crustal movement, magmatic intrusion, migration of ore-bearing fluids, erosions, etc., while geologic entities, such as lithologic phases, hydrothermal alterations, geologic structures, ore deposits, etc., are outcomes of the processes. Let $o_1, o_2, \cdots, o_k$ denote the k initial forces, $f_1, f_2, \cdots, f_p$ the p earth processes, and $z_1, z_2, \cdots, z_m$ the m geological features, including resource descriptors. Then, the cause-effect relations may be conceptualized as follows:

$$f_j = g_j\ (o_1,\ o_2,\ \cdots,\ o_k),\quad j=1,2,\cdots,p \qquad (1a)$$

$$z_i = h_i\ (f_1, f_2, \cdots, f_p),\quad i=1,\ 2,...,\ m \qquad (1b)$$

The conceptual model (1) implies that the original forces are the direct causes of the crustal evolution represented by a series of geologic processes which in turn are the direct causes of the geologic features (outcomes). Since some of these geologic features are resource descriptors, such as number of deposits, quantity of endowment, etc., relation (1) states that a mineral deposit is the result of a sequence of the superimposed geologic processes. The functions g_j's and h_i's may be assumed to be random, which provided that the original forces or geologic processes are considered to be stochastic.

A relevant question in statistical estimation of resources concerns basic statistical models useful for de-

scribing inherent relations between the geodata and resource descriptors given that geoscience information is stochastic. One should keep in mind the basic cause−effect relations (1) and these cause−effect relations do not imply any cause−effect between the resource descriptors and other geological features, although syngenetic or parallel relations do exist because both of these are outcomes of some common earth processes. For example, both argillic alteration and copper mineralization result from the same process of magmatic intrusion. Since current knowledge of the original forces is very limited, it is not realistic to discover relations g_j's in (1). Assuming that the random portions of the earth's processes can be isolated from the deterministic part, the following two sets of auxiliary relations should be essential:

$$r_l = \varphi_l(f_1, f_2, \cdots, f_p) + v_l, \qquad l=1,2,\cdots,d \qquad (2a)$$
$$z_i = \psi_i(f_1, f_2, \cdots, f_p) + e_i, \qquad i=1,2,\cdots,m \qquad (2b)$$

where r_l's are the resource descriptors, z_i's are other geologic features and v_l's and e_i's are the random errors. However, a further difficulty arises because our knowledge of earth processes is also limited. What one can observe in practice are only the geological features z_j's and maybe part of the resource descriptors. Although there is no direct causal relation between the mineral resource descriptors and other geologic features, their syngenetic and concurrent relations will assure some indirect information from the geologic features about the resources. Hence, the geological processes, and thus the mineral resource descriptors, can be mathematically reconstructed through a reverse functional estimation:

$$f_j = \Psi_j(z_1, z_2, \cdots, z_m) + \omega_j, \qquad j=1,2,...,p \qquad (3a)$$
$$r_l = \Phi_l(f_1, f_2, \cdots, f_p) + \varepsilon_l, \qquad l=1,2,\cdots,d \qquad (3b)$$

where ω_j and ε_l are the random error terms for the geological process and resource descriptor estimates.

Accordingly, if m is much greater than d, a feasible solution for mineral resource estimate may be completed in two steps: ① Factor out the f_1, f_2, etc. from relations (3a) based upon the known information on the geological features z_i's; ② Substitute these estimates of the factors into relations (3b) and derive the

estimates for the multivariate resource descriptors.

The first step of the manipulation is exactly analogous to factor−type analysis, constructing significant geologic factors(causes) from observable geological features, whereas the second step is regression−type analysis, predicting the resource descriptors (effects) from the geological factors. Consequently, factor−type and regression−type models should be fundamental multivariate statistical models for quantitative mineral re source estimation, and other relevant statistical methods may be considered as variations and combinations of the two types of method. That's why the mineral resource descriptors(r) can be statistically estimated through the geological features by the following function:

$$r_l = \theta_l(z_1, z_2, \cdots, z_m) + \theta_l, \quad l=1,2,\cdots, d \qquad (4)$$

where θ_l the random error. The geological processes are directly created by the initial forces of earth movement, while accumulation of mineral resources is directly resulted from complex interactions of the geological processes. Since the geological processes cannot be directly measured, they must be reconstructed by observable geological features, which can be, in turn, indirectly used to estimate mineral resource descriptors through relation (4).

3 Scarceness, rareness, and exceptionalness

The activities of mineral exploration have been motivated chiefly by economic and social pursuits [1]. Constantly growing economic and social demands require greater amounts of raw material, including nonrenewable mineral commodities. The conduct of mineral resource exploration is predicated upon the economic rent expected from the discovery of new deposits. An increase in the price of a mineral product, which is equivalent to the sum of the marginal rent and marginal extraction cost, indicates that the mineral resource has become scarce. A basic perspective of both geologists and economists is that mineral resources are scarce materials in the crust as they occupy only an insignificant portion of crustal material.

Any major ore deposit may be regarded in principle as an anomalous or rare phenomenon commonly

characterized by one or more geological, geochemical, and geophysical features. Consequently, signatures of significant endogenic mineralization are anomalous and exceptional geologic settings[17]. In particular, the formation of a giant deposit is an extremely rare event created by an exceptional combination of earth processes. Rareness of the giant deposits is reflected in both spatial and temporal dimensions. Significant concentrations of a metal usually have a strong affinity or correlation with particular geologic formations and epochs, as well as metallogenic environments. The genesis of giant deposits may be controlled by particular regularities that differ from those controlling the formation of medium and small-size deposits of the same composition. It is also thought that the formation of huge deposits appears to be controlled by a so-called "ore-controlling structure"[18].

Giant deposits often dominate reserves and production. It is not uncommon for a few supergiant and giant deposits to constitute over 50% of the total metal recoverable under current economic and technological conditions; accordingly, the metal quantity in small size deposits is almost negligible[19]. Conversely, giant deposits typically constitute an insignificant part of the total number of ore deposits.

Thus, the scarcity of a mineral resource is essentially determined by the fact that few giant deposits exist in the crust, but the few that do exist strongly dominate reserves and production. Accordingly, the economic viability of mineral exploration is strongly predicated upon its capability of locating the giant or large mineral deposits through delineating the associated geologically anomalous regions of the crust. Unfortunately, conventional quantitative techniques employed have failed to deal with these important particulars satisfactorily, mainly owing to inability to capture the nature of these exceptional constraints, since these unique deposits rarely exhibit common statistical properties.

The discovery process for some deposit types, e. g., those for which structural, geochemical, alteration, or geophysical signatures are correlated to deposit size or those for which discovery is primarily by drilling and for which size is strongly related to areal extent, is size biased, meaning that large, high-grade deposits tend to be discovered in early stages of the exploration of regions[6,20]. For such deposit types, the prognostication of exploration outcomes or the estimation of additional resources in undiscovered deposits should take into account the implication of this bias to the tonnages and grades of the undiscovered deposits. However, representing the discovery process of other deposit types, such as vein deposits with great vertical extent or those for which size is only weakly related to exploration anomalies, as size bias sampling may not be appropriate[21]. Improvement in locating deposits or in estimating probabilities for their occurrence requires consideration of the exploration effect and the conjunction of improved genetic, tectonic, and other unifying geoscience theories with improved synthesis methods for the effective extraction of information from diverse geodata and improved quantitative models for inference or estimation.

Considering the low concentration of many elements, e.g., 65×10^{-6} for copper, in common crust rock, the presence of a large accumulation (1-10 Mt for copper) of metal at concentrations that are mined today requires enrichments by 100s or 1000s times crustal concentrations and the accumulation of metal from a large amount of common crustal materials into a relatively small volume. Typically, this concentration or accumulation is seen as requiring the successive operations of several enrichment-depletion stages. Since these subprocesses rarely take place at the scale and strength required to form an ore deposit, their joint (sequential) occurrence could be an extremely rare event in both space and time. If each of these processes is assumed to be stochastic, the mineralization process is also stochastic, and thus the formation of ore deposits is deemed to be a rare, random event. To the extent that this assumption is acceptable, the concept of rareness of ore deposits is equivalent to the smallness of the probability for the formation of an economic deposit.

The concept of rareness can be compared to that of exceptionalness described by Gorelov[17] and the conditional exceptionalness proposed by Pan[22]. Some other terms found in literature carrying similar meanings include atypicality, uniqueness, anomaly, etc. The concept of exceptionalness is important and useful in quantitative mineral exploration. The most general feature of major commercial ore deposits is that the geological structures of their ore fields are exceptional and anomalous compared with those of neighboring areas[17].

It is noted that scarceness is a term relevant to economic aspects of resources, rareness is more closely associated with statistical(probabilistic) characteristics of mineral occurrences; and exceptionalness should be used in a geological context. More specifically, one would say that ore deposits are probabilistically rare and geologically exceptional, even though the metal derived from them may not be scarce in the economic sense described by Barnett and Morse[23]. These terms are often used to describe the status of mineralization events in a relative sense, but they can be statistically quantified in a rigorous framework.

4　Intrinsic geological unit

Most traditional resource estimations have been made on the basis of regular inter−grids or cells as the sampling scheme and estimation unit. The "cell" approach is associated with a number of drawbacks. The most significant problem is that geological processes can be reconstructed through observable geoscience features, which are measurable in geological units, not artificial cells. The cell−based measurements tend to distort the intrinsic relations between geological features and mineral resource descriptors. Secondly, quantification of the geological features, spatially correlated and even connected, is difficult to capture essential genetic factors that played key roles of metal enrichment. Finally, the cell−approach easily ignores exceptional conditions for formation of large deposits, which cannot be readily quantified through grids.

4.1　**IGU definition**

In contrast with a population of cells having multiple attributes, consider a population in which each member consists of a set of genetically related objects, e.g., igneous intrusives and associated altered host rock, and each member is described by fields of the related geologic objects. Here, mineral resource descriptors and geoscience measures are attributes of a group of geoscience fields which in turn are attributes of a set of genetically related geologic bodies. Such a scheme employs a sampling reference for quantification and integration of geoscience information that is intrinsic to the deposit type being sought. That is why the Intrinsic Geological Units (IGU) was proposed by Pan[22] and Harris and Pan[24].

The concept of intrinsic geological units, formally documented in Pan et al[25], has evolved from the notion of intrinsic samples (IS), or consistent geological area. The basic ideas behind both notions are identical and a minor difference lies in the procedure for delineation.

An appropriately delineated IGU is at once a great improvement over the traditional inter−grid area or cell because it represents the joint occurrence of geologic bodies that are genetically related to the mineral resources of interest. Thus, even before geological attributes of the IGU are quantified, the very presence of an IGU implies highly significant geoscience information about geology and mineral resources. In contrast, the cell is simply a geometric reference. Therefore, it is inevitably true that geological attributes of an IGU carry far more geoscience information than do the geological attributes of a cell.

IGUs may be formally defined as members of a population consisting of sets of genetically related geologic objects that are usually defined by their geofields[22]. Each member(IGU) of the population of IGUs constitutes an independent set of geologic objects that are genetically related to each other and to mineral deposits, although generally only some of these members contain ore deposits and mineral resources. Moreover, although a particular member of a population of IGUs contains mineral deposits, it may not be uniformly mineralized everywhere within its volume. In other words, a mineral resource unit generally is a subset of

an intrinsic geologic unit.

4.2　**Critical genetic factor**

Any mineral deposit or mineralization can be considered as an anomalous concentration of one or more elements or their chemical compounds when compared to crustal materials. This anomalous region originated from anomalous genetic processes or their superposition during certain geological epochs. Usually, a genetic model consists of a hierarchy of earth processes—from preconditions to post mineralization preservation—which acted during one or more previous time spans, and as such, these processes are not observable. Instead, the geologists must infer their previous existence and operation using observable indirect evidence, e.g., geologic features, geochemical suites, hydrothermal alteration, aeromagnetic and gravity anomalies, etc.

Since particular genetic processes were initiated and developed under certain specialized circumstances, existence of mineralization, as a significant outcome of the processes, must also be conditional upon these relevant circumstances. In other words, whether an anomalous concentration of a metal exists in a region depends solely upon the existence of certain necessary conditions during crustal evolution. Although there might exist a number of such necessary conditions for a particular genetic process or mineralization, one, or at most a few of them, is referred to as critical. For convenience, this(these) critical or necessary condition(s) is called the Critical Genetic Factor (s) (CGF). The idea of CGF does not rest solely upon one factor being more important or more critical than another in the formation of a mineral deposit, because unless all genetic factors are present, there is no mineral deposit or mineral endowment. Criticality, as used here, rests more upon the idea that the CGF arises from few, preferably only one, earth process and that those features formed by that process can be detected reasonably well by conventional sensing technologies, e.g., magnetics, gravity, geochemistry, and geology mapping. If this CGF is not present, the intrinsic geological unit is considered to be absent. For example, for a

mineral deposit related to magmatic fluids, the heat source that drives intrusion may be treated as the CGF for identification of the IGUs associated with the deposits of this type. Practically, only a single CGF is necessary for identifying spatial units that are intrinsic for mineral deposits of a single genetic type, but more than one CGF may be necessary when there is more than one genetic type of interest.

An IGU can be further understood to be a member of a population consisting of sets of geologic objects genetically associated with the CGF, each set being a member of the IGU population. Individuals from the population are called known IGUs if the related CGF is directly observed, while others are unknown or predicted when the CGF cannot be observed directly, but is inferred to exist because of the presence of geologic fields related to the CGF and to recognition criteria.

4.3　**Critical recognition criteria**

The CGF often may be identified as a process, based upon geoscience; conceptually, it may be an abstraction, instead of an observable feature. In order to make the CGF concept workable in practice, a set of special geologic features which give firm evidence of the previous existence and operation of the CGF are established. Such a feature is here termed a Critical Recognition Criterion (CRC). Each of these CRCs constitutes a sufficient condition for existence of the CGF. Any spatial location at which one or more CRCs occurs is by definition a location within an intrinsic unit.

Although the concepts of CRC make it possible for identification of CGF, the occurrence of CRCs known at the time of application may not represent the entire picture of a CGF. In other words, estimation of the presence of a CGF based upon only CRCs could be biased due to imperfect knowledge on the spatial distribution of CRCs. For example, a CRC might exist underneath the sedimentary cover, even though it is not found by surface geological mapping. This fact dictates that the identification of CRCs beyond surface observation is an important step the ap-

propriate prediction of the distribution of the CGF. This can be done by establishing statistical relations of each CRC to a set of selected geological, geochemical, and geophysical fields, which provide indirect evidence for the presence of the CGF.

Although the existence of a recognition criterion at a spatial location almost surely indicates that the location is within an IGU, the boundary of the IGU still is unknown. Consider, for example, the outcrop of a Tertiary intrusive assumed to be a CRC. Then, the outcrop area is surely within an IGU, but probably, some of the area around the outcrop also is within the same IGU because of the likelihood that at depth the intrusive extends laterally underneath the surface rocks. Consequently, the boundary of an IGU is usually uncertain. One way of representing such uncertainty is to assign each spatial location a probability for presence of one or more recognition criteria based upon a collection of geological observations at that location.

4.4　IGU delineation

At a known location　(with at least one observed CRC), the probability for the CGF should be one or very close to one. This implies that the point is almost surely within an IGU. At an unknown location (with no observed CRCs), all of the CRC probabilities estimated from geoscience fields will provide a measure of the likelihood of the presence of the CGF .

Several methods have been proposed and employed for delineating IGUs. One such example is that which consists of three steps developed by Pan et al[25]. The method delineates IGUs by estimating and combining probabilities of CRCs. Another example is given by Pan[22] and Harris et al[4] based on the union of marginal field anomalies. As discussed, the presence of a CRC gives evidence for the existence of an IGU; delineation of the boundary of the IGU is made by resolution of the geoscience fields associated with the CRCs. In this approach, the key step is to establish a procedure to identify the anomalies in terms of CRCs for each geosciences field. These anomalies (called marginal anomalies) are then combined into one

anomaly through spatial union. This is similar to the concept of using the maximum CRC probability to represent the probability for CGF.

As we know, genetic theories are most useful for grass −roots exploration or reconnaissance programs, where deposit information is not abundant. Without the guidance of genetic models, it is unsafe to select an area for a massive investment. Hence, the concept of IGU is most useful for regional mineral exploration, because it provides a quantitative framework for delineation of those areas having the conditions necessary for the presence of deposit. In large−scale exploration, such as deposit or district scale, the methodology of IGU is still useful if detailed aspects of deposit genetic models can be specified. With abundant occurrence information, it is possible to extract genetic factors as necessary conditions for the localization of deposit. However, in most cases, this detailed information is not available or not in a usable form. In general, a mining district is already a known IGU defined by broad genetic models. Unless refined genetic models are available, IGU will not provide additional power to identify areas for the potentials of deposit or district scale.

4.5　Relations between IGU and mineral target

As discussed, CGF serves as the necessary condition for presence of an IGU, but it is not a sufficient condition for the boundary definition of the IGU. The purpose of IGU proposal is to improve methodology of target identification and delineation, which, in turn, improves the effect of mineral resource assessment. The IGU theory creates a new platform on which new approach to mineral target identification can be constructed. A critical question to ask would be what is the relation between IGU and mineral targets?

5　Economic truncation and translation

Mineral deposit is not a purely geological concept when it is linked to resources and reserves.　The effects of economic truncation and translation on mineral deposits have been recognized in several decades ago, and a thorough discussion of these has been given

by Harris[3]. These phenomena reflect an important fact that mineral resources generally are a dynamic function of relevant economic and technologic constraints, including price of product and costs associated with various production phases, such as mining, milling, smelting, as well as refining. Available data on mineral deposits generally are truncated by a cost surface which is defined in terms of physical features of the deposits and technological states. In other words, the collection of mineral deposits reported reflects only the truncated fraction of the entire population of mineral deposits. Thus, use of these data directly unavoidably results in biased estimates of mineral resources, as the characteristics of the resource distribution derived from the partial data set only are a distorted representation of deposits as they occur in nature.

Translation refers to the fact that commonly reported deposit grades and tonnages are for ore reserves and that these tonnages and grades generally differ from those for the total mineralized material for the deposit as a geologic phenomenon. For deposit types having great lateral or vertical gradation in mineralization, economic rents may lead to the selection of a cutoff grade that leaves part of the deposit in the ground. When this is the case, reported ore tonnage is smaller than deposit tonnage and average grade is higher than deposit average grade.

The importance of translation as a distortion varies with the mineral commodity and the maturity of the exploration activity. In general, the greater variation of the grade within a deposit (intra deposit grade variance), the stronger the translation effect, and vice versa. For those deposit types having sharp boundaries or a uniform grade distribution, the translation effect may be negligible. For some deposit types, it is also true that the longer the deposit has been mined, the greater the reserve additions and the more representative the revised ore tonnage and grade data are of the geologic deposit. The truncation and translation effects are related to some degree when production costs are strongly influenced by ore tonnage and ore average grade, provided that intra de-

posit grade variation and the spatial distribution of grades permit the effective use of cutoff-average grade relations to maximize the net present value of economic rents. However, translation occurs mainly in mine development and subsequent mining, while truncation reflects both exploration and mining.

In order to resolve these difficulties, Harris[3] suggested a possible remedy: treating the truncation effect requires first identifying the truncation relationship, and second the explicit consideration of this relationship in the estimation of parameters, one of which is the correlation of deposit tonnage with grade. Although several attempts have been made to mitigate the difficulty in practical studies by employing more sophisticated mathematical methods in mineral endowment estimation, the problem remains to be explored further, as estimation of the cost relation is still based on the truncated data.

The importance of truncation and translation effects on a quantitative estimate of mineral resources depends to some degree upon the means of estimation and upon the objective of the estimation. For example, when estimation is to be done using analogue or control regions and the objective is to estimate the magnitude of resources for price, cost, and technology similar to those of the analogue regions, the effect of truncation and translation on the estimate may be minor. But, when the objective is to estimate the magnitude of resources for improved exploration and production technology, the effect of truncation and translation upon the estimate may be very significant.

6　Information synthesis

The geologist's view of an ore deposit may differ from that of the economist. Economists tend to consider an ore deposit as being a continuous geologic phenomenon that is discretized by applying a set of economic regularities, while geologists tend to perceive a deposit to be a discrete geologic phenomenon with anomalous concentration of one or more valuable elements[5]. Physical mechanisms of ore genesis suggest that the continuity of ore concentration is meaningful

mainly in a relative sense. A high magnitude of element concentration in host rocks often contrasts sharply with concentrations in surrounding wall rocks. This perspective may be partially illustrated by the DeWijs′ scheme of element enrichment in a deposit, which was extended by Brinck[7] to describe element concentrations within the crust. Another well−known hypothesis is Skinner′s bimodal proposition of element distribution which asserts that a gap exists between the grades of mineralized rock and the grades of common crustal material[26].

6.1 Spatial continuity

Although the continuity of the statistical distribution of grades seems to differ conceptually from that of spatial and temporal distributions, they are in fact closely related. For example, if the proposition is accepted that the grades of an element are continuously distributed in space and time, the continuity of the statistical distribution of these grades can be automatically invoked in certain environments, and vice versa. This assertion may be explained by the requirement that samples must be taken in a uniform and regular manner from the population of interest.

Metallogenic and tectonic studies depict elements to be concentrated in geologic terrains of different scales, such as ore shoot, ore body, ore district, ore belt, ore province, etc[19]. This hierarchical structure of ore formation seems to indicate that continuity exists within each of these scales, while discreteness of ore concentrations can be seen between these different scales. For instance, an ore district may be viewed as a continuously anomalous region within an ore belt, but the individual deposits included in that same district are discrete geological phenomena. This perspective carries strong implications as to sampling procedures and the organization of data for the estimation of mineral potentials.

Thus, a specific mineral exploration project focused upon the ore deposits of certain valuable elements formed and confined in a particular dimensional scale requires an appropriate sampling scheme of that same scale. For example, a new ore body developed within a deposit may be considered as mineral potential at the deposit scale, while a new ore deposit discovered in a district is regarded as mineral potential at a district scale. When estimation is aimed at predicting the mineral potentials at the district scale, the sampling scheme must accommodate the geological and mineral continuity at the corresponding hierarchical level.

6.2 Information enhancement

Although in one sense considerable progress is apparent in the use of quantitative techniques for mineral exploration and resource estimation since the early work in the 1950′s and 1960′s[27−28], much less success has been made in creating estimates that are or have been used in mineral exploration and mineral policy decisions. Even though quantitative estimation of local/drilling targets may require the detailed quantitative characterization of favorable geological, geochemical, and geophysical information, many explorationists still favor subjective and qualitative methods for the integration of geodata. Concurrent with these applications, mathematical methods were designed and demonstrated, but few were adopted. Perhaps, this is a natural evolution of the science of quantitative mineral exploration in terms of data integration, because geologists in general have been slow to adopt quantitative techniques. However, this reluctance is at least partly related to ineffective integration of geodata and insufficient extraction of geoscience information by quantitative models. Mineral resources cannot be satisfactorily estimated until more geoscience information is related by improved methods to mineral occurrence. Major difficulties that have hindered further development have been far from fully attacked, and some of them are even completely ignored.

A common practice in quantitative mineral exploration is to collect all relevant geoscience data available in the study region, including numerical observations, digitized maps, and remotely sensed images. These data are then compiled, digitized, resorted, and formatted in a readily manageable data base. Each record is usually stored as a row, while each geologic attribute occupies a column. In standard statistical

terms, each record in a data base is called a sample and each attribute is referred to as a variable. A sample in mineral exploration can be a spatial point or a one−, two−, or three−dimensional block. Most data in regional mineral exploration are interpreted in two dimensional areas.

Sampling schemes are considered to be an important factor in data interpretation and target identification. A viable sampling scheme should be able to cope with the hierarchical structures of mineralization or ore concentration. Mineralized geological bodies in different hierarchical scales correspond to different domains in space and time, which are generally defined by particular tectonic settings and geological formations. Statistically, samples should be randomly taken in the population of mineralized and nonmineralized geological blocks of the same scale. Furthermore, spatial characterization of geological features should be another criterion for reasonable representation of resource variabilities. A reliable sampling scheme should also result in a sample distribution which portrays closely the "true" population distribution of geological and mineralized bodies. Our experience has shown that quantities measured on the basis of equal area cells might lead to distorted probability distributions.

The original data may include geological, geochemical, geophysical, as well as remote sensing information in diverse modes. For example, geological data can be hydrothermal alteration, fault, and lithology, which are typically considered as nonnumerical attributes. Geochemical data can be collected from a rock outcrop, stream sample survey, or a soil grid survey. Magnetics data can be obtained from an airborne geophysical survey. It is readily seen that all these types of geodata are diverse not only in terms of sampling methods, but also the presentation of quantities. Different sampling schemes create different data densities, inconsistent spatial locations, disconnectivities, as well as uneven precisions. Different quantity presentations may give rise to even more serious problems in data integration. The most difficult problem is dealing with the correlation of different variables, which is the most critical step in geological information synthesis, especially when some data are nonnumerical. The first step in overcoming these difficulties is the quantification and unification of different data sets.

The quantification of nonnumerical attributes refers to assignment of a numerical value to each sample location; of course, the numerical value must convey explicit geological information. For example, a binary assignment gives 1 or 0 to the attributes to represent presence or absence. When each data set is "quantitative", the next step is to enhance geological information of each individual data set before they are compared, correlated, and integrated. As a matter of fact, enhancement of information from original and individual data is the most critical step towards a successful information synthesis for mineral target selection. Unfortunately, geologists traditionally tend to place too much emphasis on the original data and denigrate the importance and necessity of data filtering, cleaning, and enhancing. Conversely, some geomathematicians devote too much attention to processing of data and and give too little regard to fundamental characteristics of the original data and the useful information of the data. Original data carry the most genuine information, but they may be "contaminated" or masked by noise and even distorted due to inadequate sampling or analytical methods.

Filtering and enhancing of useful information is important to remove noise and reveal signals. Furthermore, one data set may carry information on several geological aspects. Some of these signals are not the major interests and their presence sometimes masks or distracts from the information useful in identifying mineral targets. These signal components are unwanted, even though they are not noise, and should be filtered out, or at least suppressed. However, many filtering, enhancing, and other data processing techniques can easily introduce artifacts or false signatures. For instance, a magnetic anomaly map generated from a short−wavelength filter can exhibit many high−amplitude, single −grid −point anomalies, which are known as the aliasing effect in the geophysical litera-

ture. Another example is interpolation which has been commonly used in data interpretation and quantitative mapping. All interpolation algorithms, e.g., minimum curvature and kriging, which can be considered as low pass filters, are notorious in that they tend to produce overly smoothed surfaces and quite often cause a loss of important detailed features. It is our opinion that some applications of quantitative analysis in mineral exploration have either failed to extract the important geoscience information or have created too many artifacts relative to signals; these effects are believed to be among the major reasons underlying the reluctance of geologists to replace qualitative judgment by quantitative analysis.

The above discussion suggests that filtering and enhancing is necessary for geological data interpretation and integration, but care is warranted in the use of enhancing techniques. Also, enhancement of a geological attribute includes identification and description of spatial structural characteristics, which constitute useful information about spatial auto−correlation of the attribute. More specifically, the objective of information enhancement is to maximize the signal relative to noise. By analogy, the best picture of an object taken by a camera requires a correct focus on the object; either too short or too long of a focus will blur the picture. Moreover, one should keep in mind that any enhancement technique cannot create information that is not present; instead, it is only able to reveal important features of the information carried by the attribute. But, without enhancement, some important features may not be identified nor employed in subsequent analyses. Since the amount of information in each attribute is limited, enhancement also is limited. A minimum level is necessary, for an insufficient removal of noise fails to reveal the signals to be extracted and used in subsequent analyses. Generally, the tendency of analyses is to ignore or inadequately remove noise and to over−enhance the signals. Of course, intense enhancement of data that contain noise leads to enhancement of noise as well as the signal and to false patterns and inter−relations with other information.

6.3　**Data integration**

Synthesis of geoscience information includes the quantification of geological observations, maps, and other geological images; extraction of quantitative variables; statistical preprocessing; filtering and enhancement; estimation of statistical relations among variables; and the combination of different data sets (layers). Clearly, most of the components require some amount of computation which can be performed more efficiently by using a computer. There is an obvious advantage of using a computer when many variations of the same type of analysis are required[29] or when important information includes the computer interaction of several large sets of geodata. This additional information helps to reduce uncertainties and ambiguities in geological interpretation and mineral potential estimation. Furthermore, some effective and sophisticated statistical techniques which generally prohibit manual calculations can be readily implemented on a computer.

Mineral exploration generally deals with diverse geological data in various chemical and physical forms. Appropriate information synthesis should reflect the types of information contained in each data set and their geological implications. For example, geochemical information is generally different than geophysical data. Even the same type of data, e.g., geochemical, may require different interpretation when it is obtained through different sampling techniques. For instance, soil geochemical samples are processed in different ways from stream samples. Geophysical data are rich in depth information and are capable of locating blind targets, but the extraction of such information requires appropriate processing and analysis. It is important to note that any data set has its limitations in the diagnosis of geologic favorability for mineralization, and interpretation and information synthesis must recognize these limits. Because of vast differences in geoscience content, precisions of measurement, and scales of reference among diverse geologic data, integration of these data directly can not constitute their optimum use in mineral exploration unless the data

are appropriately preprocessed and unified. Unfortunately, these problems are far less than adequately treated in traditional exploration applications.

Geoscience attributes are usually processed, correlated, and integrated to produce some estimates which characterize the favorability or probability of mineral occurrence. A more comprehensive approach treats each of the various kinds of geoscience information as a field of a particular type, e.g., geochemical fields, magnetic fields, etc.[4,24]. Mineralization may also be viewed as an ore field. The notion of field enriches useful information about three dimensional characteristics of geological bodies. Such a field is generally more expressive of meaningful geoscience information relevant to mineral resources than are "man−made" variables, e.g., measurements quantified with regard to an artificial reference, such as a grid.

A major objective of information synthesis is to maximize the extraction of relevant geoscience information in terms of mineral potentials. Geological measurements in mineral exploration are commonly multivariate in terms of either several variables (fields) measured at same sample locations, or different variables measured in different sample locations but in the same study region. In the latter case, synthesis may require an appropriate interpolation of the data before they can be jointly analyzed. When strong correlations exist among the variables, multivariate techniques are necessary to capture the joint information from multiple associations as well as the marginal contributions from individual attributes. A multivariate exploration system sometimes can be decomposed into several less significantly correlated sub systems with smaller dimensions. This partitioning may reduce the complexity of modeling and possibly permit more robust estimates at the expense of decreasing the degrees of freedom in the system.

Optimum combination of different geological data sets (layers) has been a central task in data integration and information synthesis. Agterberg[30] gives a comprehensive review on some major integration methods developed in recent years. Two major types of models notable in literature include favorability analyses and probability methods. Pan and Harris [31] propose a weighted canonical correlation method for the estimation of a favorability function. These methods are most suitable for combining continuous geological attributes. Agterberg[33] provides probabilistic techniques for combining indicator patterns in weights of evidence modeling. Both types of models, however, are deficient in some regards. Favorability methods often carry ambiguities in predicting mineral potentials, whereas evidence combination techniques are subject to strong constraints on the independency of different attributes. Moreover, as an information synthesis method, weight of evidence is simplistic. Another useful combination approach is color (RGB) image composition [33]. This type of technique also bears some serious limitations, since most current image processing software systems are only capable of combining a very limited number of "layers". Therefore, there is a need for development of more effective combination methods.

Geologic information about mineral occurrence may be roughly grouped into two categories: marginal information contributed from individual variables or fields and joint information contributed from the cross correlations between different variables or fields. The first category of information has been extensively quantified and interpreted in most of the traditional studies on mineral exploration. The second category, however, has been inadequately treated due to complexities and ambiguities. Information from the inter−dependencies of variables can be an important factor in improving the definition of exploration targets, if single exploration variables are ambiguous, noisy, and/or uncertain as to mineral occurrence. Thus, an effective synthesis technique must be able to efficiently quantify and extract the cross−correlation information.

Intuitively, there should exist a combination of variables in multivariate mineral exploration that is sufficient to capture the majority of useful information and at the same time to minimize the effort of manipulation. It is probably incorrect to think that more variables are always preferred. On the contrary, a large set

of data almost always contains redundant information which, if not appropriately eliminated, can result in unstable solutions and create noisy estimates. Therefore, another important problem in information synthesis is to select and refine variable sets such that redundant and trivial variables are excluded from consideration.

6.4　Target delineation

Mineralization is considered as an anomalous geologic event, because the element is either present in anomalous grades, rare minerals, or in anomalous quantities. The purpose of mineral exploration is to locate economic mineral deposits in such anomalous regions based on direct and most often indirect information(chemical, physical, structural, etc.) and ore genetic theories. Since the direct information, e.g., the concentration of the metal of interest, is usually meager in the early stages of exploration, indirect information (e.g., geological, geophysical, geochemical, remote sensing, etc.) is commonly employed to identify mineral exploration targets. However, the mineralized anomalies, which are distinctive from the surrounding areas in terms of the accumulated metal(s), are typically fuzzy or ambiguous in terms of indirect information. Therefore, ambiguities of information raise an intricate question, i.e., how to `best´ define targets in terms of the maximum inclusion of mineralized rock and exclusion of non−mineralized rock.

Information synthesis produces either a set of processed(enhanced, quantified, integrated) geological, geochemical, geophysical fields, or a single synthesized index characterizing the favorability/probability of mineral occurrence. Based upon the derived grids, maps, or images, all of which are commonly referred to as "layers", mineral exploration targets can be delineated by overlaying or combining the different layers. Since the synthesized results, however, are generally continuous, some threshold values are necessary to define the boundaries of targets. The traditional approaches to determine the boundaries are generally subjective and tend to introduce too many uncertainties. Obviously, a precise definition of a target is an important exploration problem to be solved.

Delineation of potential mineral targets has been a central task especially in the earlier phases of a mineral exploration program. Target areas have been identified by either subjective or objective analysis. Subjective methods provide opportunity for the maximum use of genetic theories of ore deposits and connect genetic knowledge and geological observations either intuitively by expert geologists or formally by a computer system[34−37]. Subjective methods have been generally formulated as follows: (i) formulate genetic models, (ii) relate geological observations to genetic processes, and (iii) estimate subjective probabilities of mineral occurrence. Objective (mathematical) methods attempt to maximally use various existing mineral occurrence data and quantified geological variables[28,38−42]. An objective approach generally consists of three major steps: (i) quantification of geological variables, (ii) estimation of mathematical models, and (iii) extrapolation of the estimated models to identify target areas.

Ore genesis models are crucial in mineral exploration and resource evaluation. Since genetic models of ore deposits are usually constructed on the basis of man´s past experience, imagination, and logical inference, they have a natural connection to subjective probability analyses and expert systems, giving such an approach great potential for prediction. However, in practice this approach also is subject to some limitations. First, expert systems are costly to build and to validate; second, the full potential of such systems requires the construction and incorporation of extensive data bases. Without such data bases, estimates may be associated with large uncertainties. Furthermore, genetic models change as knowledge is acquired and geologists often disagree on at least some points of a genetic model; this creates uncertainty about the identification of mineral targets. An obvious advantage of objective methods is the production of relatively robust estimates of mineral potentials by extensively using geological, geochemical, and geophysical data. However, these methods also are deficient in some regards. Without using genetic theories, geoscience information content of the variables may be low and may have poor

predicting power, i.e, the estimates often `at best´ reproduce what an expert geologist had recognized.

A useful procedure as a link between the two types of model is outlined as follows. First, based upon genetic theories, identify one or more critical genetic factors which are considered as necessary conditions for ore formation. A mineral deposit is believed to be absent if these genetic factors do not exist. Second, identify a set of recognition criteria that offer `almost sure´ existential evidence for critical genetic factors. Third, estimate the favorabilities or probabilities of occurrence of these recognition criteria based upon multiple geodata sets. Fourth, generate a synthesized favorability or probability measure for the occurrence of critical genetic factor(s) based upon the probabilities estimated in the third step. Finally, potential exploration targets are delineated from the synthesized favorability or probability measure through optimum discretization[43]. These targets have been referred to as intrinsic geological units with respect to the chosen critical genetic factor(s)[44]. These targets are so-called chiefly because they are not delineated directly in terms of mineral deposits, but in terms of the critical genetic factor that is a necessary condition for formation of the mineral deposits.

Upon the completion of target delineation, a decision needs to be made as to which targets should receive high priority to be drilled, as different targets vary in the degrees of favorability of mineral occurrence. This need requires the ranking of the targets in the sequence of drilling plans. Rank estimates may be derived directly from the synthesized fields or index. When a reasonable amount of known information on the metal(s) of interest is available in the study region, the rank estimation can be substantially improved by using a functional relation between the synthesized index and the quantity of metal.

7　Prediction with dynamic control samples

Most conventional resource analyses are constructed on the basis of extrapolation of some mathematical relations established in control areas into unknown areas[6]. Control areas are commonly employed in geodata integration and for the estimation of mineral resources of a relatively unexplored region. As such estimation is predicated upon the principle of analogy, the properties of the estimates are heavily reflective of ① how good of a geological analogue the control area is of the unexplored region and ② the economic reference for the estimated resources. When analogue and desired resource estimate is for economic and technologic conditions similar to those that induced the exploration and resource development of the control area, resource estimates produced by a mathematical model estimated on a control area may be unbiased. However, when economic or technologic references for the estimates differ or when the control area is not a good geologic analogue, resource estimates are biased and even totally wrong.

Two different approaches to improvement of estimation by mathematical models estimated on control areas are: ① use only control areas that are exhaustively explored and ② extend the mathematical model to include exploration variables (such as those defined in Pan et al[2]). Both of these solutions present difficulties however: ① except for very small regions, there are few regions large enough to make good control areas that are exhaustively explored and ② information on exploration activities generally is not available for regions large enough to make good control areas. When exploration variables are not explicitly included in the model, identification of an appropriate control area presents a difficult problem, for it must represent an unbiased sample of deposit occurrence and nonoccurrence for the relevant geologic environment. As noted by Chung et al[20], to compute unbiased estimates of the probability for deposit occurrence conditional upon a set of geologic attributes, it is necessary to know not only the distribution of various attributes in and near mineral deposits, but also the distribution of the same attributes away from mineral deposits[45].

Given the issues presented above, it is necessary to solve the dilemma in the selection of control areas

and even method of extrapolations of these control areas into unknown regions. The nature of control areas so far is static, meaning that the control areas are fixed when a mathematical model established from these control areas is extended into unexplored regions. Clearly, this static model is hardly adequate for prediction of a large region with complex variability of geological conditions and mineralization characteristics. In other words, the mathematical model built on a basis of samples collected from a control area is only appropriate when the extrapolated areas have geological conditions identical to those in the control areas. It is deemed invalid when the geological conditions in the estimated areas differ from those in the control areas. Hence, a new concept is proposed here: dynamic control areas, which are characterized as self-improvement of the mathematical models through information gains of extrapolated areas away from the initial control areas. The methodology of dynamic control areas and extrapolation of mathematical models are implemented in three steps as follows:

(1)Select the best explored areas in the working region as the initial control area, from which control samples are collected. On the basis of this sample data, a mathematical model is established data enhancement, combination of different datasets, and techniques of information synthesis. This mathematical model is then used as the initial model for extrapolation and prediction of unknown areas in the working region.

(2)Update the mathematical model when the model is used for prediction of an unknown unit based on an expanded control sample through addition of new information of exploration variables and target variables(if any) in the predicted unit. The new mathematical model will be more appropriate to the estimation of unknown units. The decision of model update is predicated upon availability of new known target variable information and variability of geological and mineralization conditions from the initial control areas.

(3)Tests are performed with the updated model with respect to its effect in prediction of known units in the initial control areas and the unknown unit. The updated model would be accepted if the test results are satisfied; otherwise, the models will be reconstructed. Quantification of variability of geological and mineralization conditions in the unknown units plays a key role in the predicting power of the updated mathematical models.

The model update above is in nature an iterative process, which improves predictability of the model in the unknown units. The initial control sample is only used for establishment of the initial mathematical model, which is then updated and optimized as it is extended into the predicted areas through incorporation of new information on the variability of geological environments.

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